Homework 1
Math 117 - Summer 2022

1) (5 points) Let ∞ and $-\infty$ be two objects, not in $\mathbb{R}$. We will define an addition/scalar multiplication on the set $R \cup \{\infty\} \cup \{-\infty\}$ by

$$t\infty = \begin{cases} -\infty & t < 0 \\ 0 & t = 0 \\ \infty & t > 0 \end{cases} \tag{1}$$

$$t(-\infty) = \begin{cases} \infty & t < 0 \\ 0 & t = 0 \\ -\infty & t > 0 \end{cases} \tag{2}$$

$$t + \infty = \infty + t = \infty \tag{3}$$

$$t + (-\infty) = -\infty + t = -\infty \tag{4}$$

$$\infty + \infty = \infty \tag{5}$$

$$-\infty + (-\infty) = -\infty \tag{6}$$

$$\infty + (-\infty) = 0 \tag{7}$$

(and the sum and product of real numbers is as usual).
Is $R \cup \{\infty\} \cup \{-\infty\}$ a vector space over $\mathbb{R}$? If so, prove it (ie, verify all the axioms); if not, show explicitly which axiom fails.

Solution:

2) (3 points) Consider the field $\mathbb{F}_{17}$ and the list of three vectors in $\mathbb{F}_{17}^3$ given by:

$$S = \left(\begin{pmatrix} \overline{7} \\ \overline{8} \\ \overline{1} \end{pmatrix}, \begin{pmatrix} \overline{0} \\ \overline{2} \\ \overline{3} \end{pmatrix}, \begin{pmatrix} \overline{12} \\ \overline{13} \\ \overline{16} \end{pmatrix} \right)$$

Are these vectors linearly independent in $\mathbb{F}_{17}^3$ (Hint: the way you determined this in Math 21 works here, just remember to work mod 17)

Solution:

3) Let V be a vector space over any field $\mathbb{F}$:

(a) (5 points) Prove that V is infinite dimensional iff there is a sequence of vectors $(v_1, v_2, v_3, \dots)$ in V such that $v_1, v_2, \dots, v_m$ is Linearly independent for each positive integer m.

(b) Use the above to conclude the following two vector spaces are infinite dimensional $\mathbb{C}$-vector spaces:

  (a) (1 point) $V = Fct(\mathbb{Z}, \mathbb{C})$

  (b) (1 point) $V = \mathbb{C}[t]$ (the vector space of all polynomials)

Solution:

4) (5 points) Let V be a vector space over some field $\mathbb{F}$

(a) (3 points) Let W be a subspace of a vector space V. Suppose $x, y \in V$ are not in W. Show that if $x \in span(W, y)$ then $y \in span(W, x)$

(b) (2 point) Give an example of a vector space V, a subspace $W \subseteq V$, and vectors $x, y \in V$ such that $x \in span(W, y)$ but $y \notin span(W, x)$

Solution: